

國立高雄應用科技大學
101 學年度碩士班招生考試
土木工程與防災科技研究所

准考證號碼 (考生必須填寫)

工程數學 (甲組)

試題 共 2 頁, 第 1 頁

注意: a. 本試題共 7 題, 共 100 分。

b. 作答時不必抄題, 但請標明對應題號, 如: 1 (a), 1(b), 2, 等。

c. 考生作答前請詳閱答案卷之考生注意事項。

1. Given an ODE: $P(y) dx + (2y + x \cos y) dy = 0$,

(a) if this equation is an exact ODE, $P(y) = ?$ (5%)

(b) using (a) solve the general solution in a form of $F(x, y) = C$. (5%)

2. If the complete solution of 2nd-order ODE with constant coefficients

$$P, Q, R: \frac{d^2 y}{dx^2} + P \frac{dy}{dx} + Q y = R e^x$$

is $y(x) = C_1 e^{3x} + C_2 e^{-2x} - \frac{1}{6} e^x$, solve for the values of P, Q, R . (10%)

3. Solve the general solutions of ODE: $\frac{dy}{dx} = -\frac{y}{x} + 3\sqrt{\frac{x}{y}}$ (10%)

4. Given two vectors $\vec{a} = (1, 1, 1)$ and $\vec{b} = (-1, 0, 1)$.

(a) What's the angle between \vec{a} and \vec{b} ? (10%)

(b) Find unit vectors \vec{e}_c of vector \vec{c} which are perpendicular to \vec{a} and \vec{b} simultaneously, i.e., $\vec{c} \perp \vec{a}$ and $\vec{c} \perp \vec{b}$. (10%)

(c) Evaluate $|\vec{a} \cdot (\vec{b} \times \vec{c})| = ?$ (10%)

(To be continued)

[後續尚有試題]

5. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$, Evaluate $I = \int_0^{2\pi} [xdy - ydx]$ (10%)

6. Equation of a two-degrees-of-freedom dynamic system

$$(-\omega^2[M] + [K]) \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where $[M] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $[K] = \begin{bmatrix} a & -1 \\ -1 & b \end{bmatrix}$ and a, b are positive constants.

(a) If the two eigenvalues are $\omega_1 = 1, \omega_2 = \sqrt{3}$, $a=? b=?$ (10%)

(b) Solve for the eigenvectors \vec{X}_1, \vec{X}_2 corresponding to these two eigenvalues.

(10%)

7. The constitutive relationship of a new material is expressed as

$$\{\sigma\} = [C] \{\varepsilon\} - [e]^T \{E\}$$

$$\{D\} = [e] \{\varepsilon\} + [\epsilon] \{E\}$$

where $[C], [e], [\epsilon]$ are constant matrices, $\{\sigma\}, \{D\}, \{\varepsilon\}, \{E\}$ are physical variables. If matrix $[C]$ is non-singular, this can also be expressed in another form as

$$\{\varepsilon\} = [P] \{\sigma\} + [Q] \{E\}$$

$$\{D\} = [R] \{\sigma\} + [S] \{E\}$$

Please try to express $[P], [Q], [R], [S]$ in terms of $[C], [e]^T, [e], [\epsilon]$.

(Note: $[]$ denotes matrices, $\{ \}$ denotes vectors.) (10%)

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